Extending the Standard Model to Include CPT- and Lorentz-Breaking Terms

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Low-energy remnant fundamental symmetry violations may be present in nature at levels attainable in upcoming experiments. These effects may arise through spontaneous symmetry breaking in a more complete Lorentz covariant theory underlying the standard model. In this work the issue of parameterizing such violations in a consistent and complete manner is addressed. The approach is to use the mechanism of spontaneous symmetry breaking to generate all possible terms consistent with gauge invariance and power-counting renormalizability to construct an extension of the standard model that includes Lorentz- and CPT-breaking terms. A consistent quantization of the theory is developed, conventional quantum field theoretic techniques are shown to apply, and some ramifications for quantum electrodynamics are explored.

1 Introduction and Motivation

Lorentz invariance serves as a fundamental guiding principle of virtually every theory describing fundamental particle interactions. When applied to local, point-particle theories, the assumed Lorentz invariance coupled with some mild technical assumptions leads one to conclude that CPT must be preserved in the theory.[1]

However, if the fundamental theory underlying the standard model is constructed using nonlocal objects such as strings, Lorentz symmetry may not be exactly preserved in the low-energy limit of the full theory which is assumed to contain the standard model. An explicit mechanism of this type has been proposed whereby spontaneous Lorentz symmetry breaking may occur in string theory.[2,3]

Our approach here is to use the mechanism of spontaneous symmetry breaking to generate a list of possible Lorentz violating interactions between standard model fields. The standard model extension is constructed by selecting those terms satisfying SU(3)×SU(2)×U(1) gauge invariance and power-counting renormalizability.[4] By only using the property of spontaneous symmetry breaking and not referring to explicit details of the underlying theory, we are able to construct a very general model of Lorentz breaking in the context of the standard model.

High precision measurements involving atomic systems, [5,6] clock comparisons, [7] and neutral meson oscillations [8] provide stringent tests of Lorentz and

CPT symmetry. They are treated in great detail elsewhere in these proceedings. The implications of CPT-violating terms of the type described in this work on baryogenesis have also been investigated.[9]

2 Lorentz Violation via Spontaneous Symmetry Breaking

Conventional spontaneous symmetry breaking occurs in the Higgs sector of the standard model where the Higgs field gains an expectation value, thereby partially breaking SU(2)×U(1) gauge invariance. This happens because an assumed potential for the Higgs field has a minimum at some nonzero value of the field.

For example, given a simple Lagrangian describing a single fermion field ψ and a single scalar field ϕ of the form

$$\mathcal{L} = \mathcal{L}_0 - \mathcal{L}' \quad , \tag{1}$$

where

$$\mathcal{L}' \supset \lambda\phi\overline{\psi}\psi + \mathrm{h.c.} - (\phi^2 - \mathrm{a}^2)^2 \quad , \tag{2}$$

a nonzero vacuum expectation value $\langle\phi\rangle$ for the scalar field will minimize the energy, hence generating a fermion mass of $m_f = \lambda\langle\phi\rangle$. This expectation value of the scalar field breaks the SU(2)×U(1) gauge invariance because $\langle\phi\rangle$ no longer transforms nontrivially under the gauge group. Lorentz symmetry is maintained in this case because $\langle\phi\rangle$ and ϕ both transform trivially under boosts and rotations.

Notice that if a *tensor* T gains a nonzero vacuum expectation value, $\langle T\rangle$, Lorentz symmetry will be spontaneously broken. To see how this might occur, consider a Lagrangian describing a fermion ψ and a tensor T of the form

$$\mathcal{L} = \mathcal{L}_0 - \mathcal{L}' \quad , \tag{3}$$

where

$$\mathcal{L}' \supset \frac{\lambda}{M^k} T \cdot \overline{\psi}\Gamma(i\partial)^k\psi + \mathrm{h.c.} + \mathrm{V(T)} \quad . \tag{4}$$

In this expression, λ is a dimensionless coupling, M is some heavy mass scale of the underlying theory, Γ denotes a general gamma matrix structure in the Dirac algebra, and $V(T)$ is a potential for the tensor field. (indices are suppressed for notational simplicity) The potential $V(T)$ is assumed to arise from a theory underlying the standard model. Terms contributing to $V(T)$ are precluded from conventional renormalizable four-dimensional field theories, but may arise in the low-energy limit of a more general theory such as string theory. [2]

If the potential $V(T)$ is such that it has a nontrivial minimum, a vacuum expectation value $\langle T \rangle$ will be generated. There will then be a term of the form

$$\mathcal{L}' \supset \frac{\lambda}{M^k} \langle T \rangle \cdot \overline{\psi} \Gamma (i\partial)^k \chi + \text{h.c.} \quad , \tag{5}$$

present in the Lagrangian. Terms of this type can break Lorentz invariance and various discrete symmetries C, P, T, CP, and CPT.

3 Relativistic Quantum Mechanics and Field Theory

To develop theoretical techniques for treating generic terms of the form given in Eq. (5), a specific example is studied. The example presented here involves a single fermion Lagrangian containing Lorentz-violating terms without derivative couplings ($k = 0$) that also violate CPT.

We proceed by listing the possible gamma-matrix structures that could arise in such a term:

$$\Gamma \sim \{1, \gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu}, \gamma^5\} \quad . \tag{6}$$

The condition that a fermion bilinear with no derivative couplings violates CPT is equivalent to the requirement that $\{\Gamma, \gamma^5\} = 0$. Half of the matrices in Eq. (6) satisfy this condition: $\Gamma \sim \gamma^\mu$ and $\Gamma \sim \gamma^5\gamma^\mu$. The contribution to the lagrangian from these terms can be written as

$$\mathcal{L}'_a \equiv a_\mu \overline{\psi} \gamma^\mu \psi \quad , \quad \mathcal{L}'_b \equiv b_\mu \overline{\psi} \gamma_5 \gamma^\mu \psi \quad , \tag{7}$$

where a_μ and b_μ are constant coupling coefficients that parameterize the tensor expectation values and relevant couplings arising in Eq. (5). These parameters are assumed suppressed with respect to other physically relevant energy scales in the low-energy effective theory.

Combining these terms with the conventional single fermion terms gives a model lagrangian of

$$\mathcal{L} = \frac{i}{2} \overline{\psi} \gamma^\mu \overleftrightarrow{\partial_\mu} \psi - a_\mu \overline{\psi} \gamma^\mu \psi - b_\mu \overline{\psi} \gamma_5 \gamma^\mu \psi - m \overline{\psi} \psi \quad . \tag{8}$$

Several features of this modified theory are immediately apparent. The lagrangian is hermitian, thereby obeying conventional quantum mechanics, conservation of probability and unitarity. Translational invariance implies the existence of a conserved energy and momentum. Explicitly, the conserved four-momentum is constructed as

$$P_\mu = \int d^3x \Theta^0{}_\mu = \int d^3x \tfrac{1}{2} i \overline{\psi} \gamma^0 \overleftrightarrow{\partial_\mu} \psi \quad , \tag{9}$$

just as in the conventional case. The Dirac equation that results from Eq. (8) is linear in the fermion field allowing an exact solution of the free theory. Finally, the global U(1) invariance of the model lagrangian implies the existence of a conserved current $j_\mu = \overline{\psi}\gamma^\mu\psi$.

The resulting Dirac equation obtained by variation of Eq. (8) with respect to the fermion field is

$$(i\gamma^\mu\partial_\mu - a_\mu\gamma^\mu - b_\mu\gamma_5\gamma^\mu - m)\psi = 0 \quad . \tag{10}$$

Due to the linearity of the equation, plane-wave solutions

$$\psi(x) = e^{\pm ip_\mu x^\mu} w(\vec{p}) \quad , \tag{11}$$

can be used to solve the equation. Substitution into the modified Dirac equation yields

$$\begin{aligned}(\pm p_\mu\gamma^\mu - a_\mu\gamma^\mu - b_\mu\gamma_5\gamma^\mu - m)w(\vec{p}) &\equiv M_\pm w(\vec{p}) \\ &= 0 \quad . \end{aligned} \tag{12}$$

A nontrivial solution will exist only if $Det M_\pm = 0$. This imposes a condition on $p^0(\vec{p}) \equiv E(\vec{p})$ generating a dispersion relation for the fermion.

The general solution involves a fourth-order polynomial that can be solved algorithmically, but the resulting solution is complex and not very illuminating. Here we will consider the special case $\vec{b} = 0$. In this case the exact dispersion relations are

$$E_+(\vec{p}) = \left[m^2 + (|\vec{p} - \vec{a}| \pm b_0)^2\right]^{1/2} + a_0 \quad , \tag{13}$$

$$E_-(\vec{p}) = \left[m^2 + (|\vec{p} + \vec{a}| \mp b_0)^2\right]^{1/2} - a_0 \quad . \tag{14}$$

Examination of the above energies reveals some qualitative effects of the CPT-violating terms. The usual four-fold energy degeneracy of spin-$\frac{1}{2}$ particles and antiparticles is removed by a_μ and b_0. The particle-antiparticle degeneracy is broken by a_μ and the helicity degeneracy is split by b_0. The corresponding spinor solutions $w(\vec{p})$ can be explicitly calculated, forming an orthogonal basis of states as expected.

One interesting feature of these solutions is the unconventional relationship that exists between momentum and velocity. For a wave packet of positive helicity particles with four momentum $p^\mu = (E, \vec{p})$, the expectation value of the velocity operator $\vec{v} = i[H, \vec{x}] = \gamma^0\vec{\gamma}$ is calculated as

$$\langle\vec{v}\rangle = \langle\frac{(|\vec{p} - \vec{a}| - b^0)}{(E - a^0)}\frac{(\vec{p} - \vec{a})}{|\vec{p} - \vec{a}|}\rangle \quad . \tag{15}$$

Examination of the velocity using a general dispersion relation reveals that $|v_j| < 1$ for arbitrary b_μ, and that the limiting velocity as $\vec{p} \to \infty$ is 1. This implies that the effects of the CPT violating terms are mild enough to preserve causality. This will be verified independently from the perspective of field theory that we will now develop.

To quantize the theory, the general expansion for ψ in terms of its spinor components given by

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha=1}^{2} \left[\frac{m}{E_u^{(\alpha)}} b_{(\alpha)}(\vec{p}) e^{-ip_u^{(\alpha)} \cdot x} u^{(\alpha)}(\vec{p}) \right.$$
$$\left. + \frac{m}{E_v^{(\alpha)}} d^*_{(\alpha)}(\vec{p}) e^{ip_v^{(\alpha)} \cdot x} v^{(\alpha)}(\vec{p}) \right] \quad , \tag{16}$$

is promoted in the usual way to an operator acting on a Hilbert space of basis states.

Calculation of the energy from Eq. (9) using conventional normal ordering yields a positive definite quantity (for $|a^0| < m$) provided the following nonvanishing anticommutation relations are imposed on the creation and annihilation operators:

$$\{b_{(\alpha)}(\vec{p}), b^\dagger_{(\alpha')}(\vec{p}\,')\} = (2\pi)^3 \frac{E_u^{(\alpha)}}{m} \delta_{\alpha\alpha'} \delta^3(\vec{p} - \vec{p}\,') \quad ,$$
$$\{d_{(\alpha)}(\vec{p}), d^\dagger_{(\alpha')}(\vec{p}\,')\} = (2\pi)^3 \frac{E_v^{(\alpha)}}{m} \delta_{\alpha\alpha'} \delta^3(\vec{p} - \vec{p}\,') \quad . \tag{17}$$

The resulting equal-time anticommutators of the fields are

$$\{\psi_\alpha(t, \vec{x}), \psi^\dagger_\beta(t, \vec{x}')\} = \delta_{\alpha\beta} \delta^3(\vec{x} - \vec{x}') \quad ,$$
$$\{\psi_\alpha(t, \vec{x}), \psi_\beta(t, \vec{x}')\} = 0 \quad ,$$
$$\{\psi^\dagger_\alpha(t, \vec{x}), \psi^\dagger_\beta(t, \vec{x}')\} = 0 \quad . \tag{18}$$

These relations show that the conventional Fermi statistics remain unaltered.

The conserved charge Q and momentum P^μ are computed as

$$Q = \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha=1}^{2} \left[\frac{m}{E_u^{(\alpha)}} b^\dagger_{(\alpha)}(\vec{p}) b_{(\alpha)}(\vec{p}) - \frac{m}{E_v^{(\alpha)}} d^\dagger_{(\alpha)}(\vec{p}) d_{(\alpha)}(\vec{p}) \right] \quad , \tag{19}$$

$$P_\mu = \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha=1}^{2} \left[\frac{m}{E_u^{(\alpha)}} p^{(\alpha)}_{u\mu} b^\dagger_{(\alpha)}(\vec{p}) b_{(\alpha)}(\vec{p}) \right.$$
$$\left. + \frac{m}{E_v^{(\alpha)}} p^{(\alpha)}_{v\mu} d^\dagger_{(\alpha)}(\vec{p}) d_{(\alpha)}(\vec{p}) \right] \quad . \tag{20}$$

From these expressions we see that the charge of the fermion is unperturbed and the energy and momentum satisfy the same energy momentum relations that we found using relativistic quantum mechanics.

Causality is governed by the unequal-time anticommutation relations for the fermion fields. Explicit integration for the case of $\vec{b} = 0$ proves that

$$\{\psi_\alpha(x), \overline{\psi}_\beta(x')\} = 0 \quad , \tag{21}$$

for spacelike separations $(x - x')^2 < 0$. This result shows that physical observables separated by spacelike intervals will in fact commute (for case $\vec{b} = 0$). This agrees with our previous results obtained by examination of the velocity using the relativistic quantum mechanics approach.

Next the problem of extending the free field theory to interacting theory is addressed. Most of the conventional formalism developed for perturbative calculations in the interacting theory carries over directly to the present case. The main reason that these techniques work is that the Lorentz violating modifications introduced are linear in the fermion fields. The main result is that the usual Feynman rules apply provided that the Feynman propagator is modified as

$$S_F(p) = \frac{i}{p_\mu \gamma^\mu - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - m} \quad , \tag{22}$$

and the exact spinor solutions of the modified free fermion theory are used on external legs.

4 Standard Model Extension

We now turn to the question of how to apply spontaneous symmetry breaking to generate Lorentz-violating terms using standard model fields. Our approach is to consider all possible terms that can arise from spontaneous symmetry breaking that satisfy power-counting renormalizability and preserve the SU(3)×SU(2)×U(1) gauge invariance of the standard model.[4] The relevant terms contribute to all sectors of the standard model. In listing the terms below, the Lorentz violating terms are classified according to their properties under the CPT transformation.

In the lepton sector the left- and right-handed multiplets are of the form

$$L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L \quad , \quad R_A = (l_A)_R \quad , \tag{23}$$

where $A = 1, 2, 3$ labels the flavor:

$$l_A \equiv (e, \mu, \tau) \quad , \quad \nu_A \equiv (\nu_e, \nu_\mu, \nu_\tau) \quad . \tag{24}$$

The Lorentz-violating terms satisfying the required properties are

$$\mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{lepton}} = \tfrac{1}{2} i (c_L)_{\mu\nu AB} \overline{L}_A \gamma^\mu \overset{\leftrightarrow}{D^\nu} L_B \\ + \tfrac{1}{2} i (c_R)_{\mu\nu AB} \overline{R}_A \gamma^\mu \overset{\leftrightarrow}{D^\nu} R_B \quad , \tag{25}$$

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{lepton}} = -(a_L)_{\mu AB}\ \overline{L}_A \gamma^\mu L_B \\ -(a_R)_{\mu AB}\ \overline{R}_A \gamma^\mu R_B \quad . \tag{26}$$

In the above expression $c_{\mu\nu}$ and a_μ are constant coupling coefficients related to the background expectation values of the corresponding tensor fields, and D^μ is the usual covariant derivative.

These are not the final form of the standard model terms because the SU(2)×U(1) symmetry is broken by the Higgs mechanism. Once this breaking occurs, the fields in Eq. (26) can be rewritten in terms of the physical Dirac spinors corresponding to the observed leptons and neutrinos. For example, the CPT-odd lepton terms become

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{lepton}} = -(a_\nu)_{\mu AB}\ \overline{\nu}_A \tfrac{1}{2}(1+\gamma_5)\gamma^\mu \nu_B \\ -(a_l)_{\mu AB}\ \overline{l}_A \gamma^\mu l_B \\ -(b_l)_{\mu AB}\ \overline{l}_A \gamma_5 \gamma^\mu l_B \quad . \tag{27}$$

Note that b_μ coupling coefficients arise in the process of combining right- and left-handed fields into Dirac spinors.

If we now examine the first generation electron contribution corresponding to $A = B = 1$, we find

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{lepton}} \supset -(a_l)_{\mu 11}\ \overline{e}\gamma^\mu e - (b_l)_{\mu 11}\ \overline{e}\gamma_5\gamma^\mu e \quad . \tag{28}$$

These terms are exactly the form of Eq. (7) that were analyzed in the previous section. The relativistic quantum mechanics and field theoretic techniques developed to handle these terms are therefore directly applicable to electrons. Terms in Eq. (27) of the form $A \neq B$ lead to small lepton flavor-changing amplitudes.

The construction of the extension in the quark sector is similar to that in the lepton sector. The main difference is that corresponding right-handed quark fields are now present for each left-handed field unlike the case in the lepton sector. The left- and right-handed quark multiplets are

$$Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L \quad , \quad \begin{matrix} U_A = (u_A)_R \\ D_A = (d_A)_R \end{matrix} \quad , \tag{29}$$

where $A = 1, 2, 3$ labels quark flavor

$$u_A \equiv (u, c, t) \quad , \quad d_A \equiv (d, s, b) \quad . \tag{30}$$

The Lorentz-violating terms are of the same form as in the lepton sector and will not be explicitly given here. The diagonal $A = B$ terms are again of the same form as Eq. (7) analyzed in the previous section. The quark a_μ terms are particularly interesting because they can lead to CPT-violating effects in neutral meson systems.[10]

Turning next to the Higgs sector, there are contributions involving two Higgs fields, and generalized Yukawa coupling terms involving a single Higgs and two fermion fields. The Lorentz-violating terms quadratic in the Higgs fields are

$$\begin{aligned}
\mathcal{L}^{\rm CPT-even}_{\rm Higgs} = {} & \tfrac{1}{2}(k_{\phi\phi})^{\mu\nu}(D_\mu\phi)^\dagger D_\nu\phi + \text{h.c.} \\
& -\tfrac{1}{2}(k_{\phi B})^{\mu\nu}\phi^\dagger\phi B_{\mu\nu} \\
& -\tfrac{1}{2}(k_{\phi W})^{\mu\nu}\phi^\dagger W_{\mu\nu}\phi \quad ,
\end{aligned} \tag{31}$$

$$\mathcal{L}^{\rm CPT-odd}_{\rm Higgs} = i(k_\phi)^\mu \phi^\dagger D_\mu\phi + \text{h.c.} \quad , \tag{32}$$

where $W_{\mu\nu}$ and $B_{\mu\nu}$ are the field strengths for the SU(2) and U(1) gauge fields and the various k parameters are coupling constants related to tensor expectation values.

The Yukawa type terms involving one Higgs are

$$\begin{aligned}
\mathcal{L}^{\rm CPT-even}_{\rm Yukawa} = {} & -\tfrac{1}{2}\big[(H_L)_{\mu\nu AB}\overline{L}_A\phi\sigma^{\mu\nu}R_B \\
& +(H_U)_{\mu\nu AB}\overline{Q}_A\phi^c\sigma^{\mu\nu}U_B \\
& +(H_D)_{\mu\nu AB}\overline{Q}_A\phi\sigma^{\mu\nu}D_B\big] + \text{h.c.} \quad ,
\end{aligned} \tag{33}$$

where the H parameters are related to tensor expectation values.

One interesting result of including these terms into the standard model is that the conventional SU(2)×U(1) breaking is modified. When the static potential is minimized, the Z^0 boson gains an expectation value of

$$\langle Z^0_\mu \rangle = \frac{1}{q}\sin 2\theta_W (\mathrm{Re}\,\hat{k}_{\phi\phi})^{-1}_{\mu\nu} k^\nu_\phi \quad , \tag{34}$$

where $\hat{k}^{\mu\nu}_{\phi\phi} = \eta^{\mu\nu} + k^{\mu\nu}_{\phi\phi}$, q is the electric charge, and θ_W is the weak mixing angle. Note that if the CPT-odd term k_ϕ vanishes then $\langle Z^0_\mu \rangle = 0$. This is reasonable because a nonzero value of $\langle Z^0_\mu \rangle$ violates CPT symmetry.

The final sector to be examined is the gauge sector. The various Lorentz-breaking terms satisfying our criteria are

$$\mathcal{L}_{\text{gauge}}^{\text{CPT-even}} = -\tfrac{1}{2}(k_G)_{\kappa\lambda\mu\nu}\text{Tr}(G^{\kappa\lambda}G^{\mu\nu}) \\ -\tfrac{1}{2}(k_W)_{\kappa\lambda\mu\nu}\text{Tr}(W^{\kappa\lambda}W^{\mu\nu}) \\ -\tfrac{1}{4}(k_B)_{\kappa\lambda\mu\nu}B^{\kappa\lambda}B^{\mu\nu} \quad , \tag{35}$$

$$\mathcal{L}_{\text{gauge}}^{\text{CPT-odd}} = k_{3\kappa}\epsilon^{\kappa\lambda\mu\nu}\text{Tr}(G_\lambda G_{\mu\nu} + \tfrac{2i}{3}G_\lambda G_\mu G_\nu) \\ + k_{2\kappa}\epsilon^{\kappa\lambda\mu\nu}\text{Tr}(W_\lambda W_{\mu\nu} + \tfrac{2i}{3}W_\lambda W_\mu W_\nu) \\ + k_{1\kappa}\epsilon^{\kappa\lambda\mu\nu}B_\lambda B_{\mu\nu} \quad . \tag{36}$$

In these expressions, the k terms are the background coupling constants and the $G^{\mu\nu}$, $W^{\mu\nu}$, and $B^{\mu\nu}$ are the field strengths for the SU(3), SU(2), and U(1) gauge fields respectively.

The CPT-odd terms can generate negative contributions to the energy [11] creating an instability in the theory. One option is to set these coefficients to zero, and show that they remain zero at the quantum level. This has been carried out to the one-loop level by utilizing an anomaly cancellation mechanism that must be inherited from any consistent theory underlying the standard model.[4]

5 Restriction to QED

Here we restrict our attention to the theory of electrons and photons that results from the above extension of the standard model. The usual QED Lagrangian is

$$\mathcal{L}_{\text{electron}}^{\text{QED}} = \tfrac{1}{2}i\overline{\psi}\gamma^\mu \overleftrightarrow{D_\mu} \psi - m_e\overline{\psi}\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad , \tag{37}$$

where ψ is the electron field, m_e is its mass, and $F^{\mu\nu}$ is the photon field strength tensor.

Selecting out the CPT-even electron terms that violate Lorentz symmetry from the full standard model extension yields

$$\mathcal{L}_{\text{electron}}^{\text{CPT-even}} = -\tfrac{1}{2}H_{\mu\nu}\overline{\psi}\sigma^{\mu\nu}\psi \\ +\tfrac{1}{2}ic_{\mu\nu}\overline{\psi}\gamma^\mu \overleftrightarrow{D^\nu} \psi \\ +\tfrac{1}{2}id_{\mu\nu}\overline{\psi}\gamma_5\gamma^\mu \overleftrightarrow{D^\nu} \psi \quad , \tag{38}$$

where H, c, and d are constant coupling coefficients.

The CPT-odd electron terms are

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{electron}} = -a_\mu \overline{\psi}\gamma^\mu\psi - b_\mu\overline{\psi}\gamma_5\gamma^\mu\psi \quad , \tag{39}$$

where a and b are parameters analogous to those in Eq. (7) applied to the electron field.

The photon corrections are given by

$$\mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{photon}} = -\tfrac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} \quad , \tag{40}$$

and

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{photon}} = +\tfrac{1}{2}(k_{AF})^\kappa\epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu} \quad . \tag{41}$$

The parameters k_F and k_{AF} are the appropriate linear combinations of parameters in Eqs.(35) and (36) that result when the photon is defined as the unbroken U(1) electric force mediator.

For an explicit example, we will examine a special case in which $(k_{AF})^\mu = 0$ (no CPT-odd piece), and $(k_F)_{0j0k} = -\frac{1}{2}\beta_j\beta_k$. With this choice the free photon lagrangian takes the form

$$\mathcal{L}^{\mathrm{special}}_{\mathrm{photon}} = \tfrac{1}{2}(\vec{E}^2 - \vec{B}^2) + \tfrac{1}{2}(\vec{\beta}\cdot\vec{E})^2 \quad . \tag{42}$$

Variation with respect to the dynamical fields yields the following modified source-free Maxwell equations:

$$\begin{aligned} \vec{\nabla}\cdot\vec{E} &= -\vec{\beta}\cdot\vec{\nabla}(\vec{\beta}\cdot\vec{E}) \quad , \\ \vec{\nabla}\times\vec{B} - \partial_0\vec{E} &= \vec{\beta}\;\partial_0(\vec{\beta}\cdot\vec{E}) \quad . \end{aligned} \tag{43}$$

The other two remain unmodified as they are simply a result of the definitions of $\vec{E}$ and $\vec{B}$ in terms of A^μ which have not been modified.

These equations can be solved with the plane-wave ansatz

$$A_\mu(x) \equiv \epsilon_\mu(p)\exp(-ip_\alpha x^\alpha) \quad , \tag{44}$$

because the modifications are linear in the dynamical fields. Proceeding in the Lorentz gauge where $p_\mu A^\mu = 0$ (there is no difficulty in selecting this gauge because we have maintained gauge invariance in the standard-model extension) we find that a solution exists provided p_μ satisfies one of

$$(p_o)^2 = 0 \quad , \tag{45}$$

$$(p_e)^2 = -\frac{(\vec{\beta}\times\vec{p}_e)^2}{1+\vec{\beta}^2} \quad , \tag{46}$$

where p_o denotes an ordinary mode and p_e denotes an extraordinary mode of propagation. The ordinary mode satisfies the conventional dispersion relation of electromagnetic waves and therefore behaves identically the same as ordinary photons. The extraordinary mode is more interesting because it satisfies a modified dispersion relation.

Taking the case $\beta \cdot \vec{p} = 0$ for simplicity, the ordinary mode is polarized with $\vec{A}_o$ along the direction of $\vec{p} \times \vec{\beta}$ while the extraordinary mode $\vec{A}_e$ is polarized along $\vec{\beta}$. They are both perpendicular to the momentum of the wave $\vec{p}$. The group velocities defined by $\vec{v}_g \equiv \vec{\nabla}_p p^0$ take the form

$$\vec{v}_{g,o} = \hat{p} \quad , \quad \vec{v}_{g,e} = \frac{1}{\sqrt{1+\vec{\beta}^2}} \, \hat{p} \quad . \tag{47}$$

The extraordinary mode travels with a modified velocity that is slightly less than the velocity of the ordinary mode.

A general wave will be a linear superposition of the A_o and A_e modes. The electric field that results is

$$\vec{E}(t,\vec{x}) = -p^0 \left(c_o \hat{A}_o \sin[p^0(r-t)] + c_e \hat{A}_e \sin[p^0(\sqrt{1+\vec{\beta}^2} \; r - t)] \right) \quad , \tag{48}$$

where the weights c_o and c_e are fixed by the initial polarization conditions. As the wave evolves in time, a plane polarized wave will in general become elliptically polarized after traveling a distance

$$r \simeq \frac{\pi}{2\left(\sqrt{1+\vec{\beta}^2} - 1\right) p^0} \simeq \frac{\pi}{\vec{\beta}^2 p^0} \quad , \tag{49}$$

where the approximation holds for $\vec{\beta}^2 \propto k_F << 1$. The magnetic field behaves similarly. Terms of this form have interesting implications for photon birefringence, in particular they contribute to polarization rotation from quasars.[12]

6 Summary

A framework was presented that incorporates Lorentz- and CPT-violating effects into the context of conventional quantum field theory. Using a generic spontaneous symmetry breaking mechanism as the source of these terms, an extension of the standard model that includes Lorentz and CPT breaking was developed. The extension preserves power-counting renormalizability and SU(3)×SU(2)×U(1) gauge invariance. The parameters introduced can be used to establish quantitative bounds on CPT- and Lorentz-breaking effects in nature. The restriction to QED was explored resulting in interesting implications for photon propagation.

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